The canonical effect in statistical models for relativistic heavy ion collisions

A Keränen†and F Becattini‡

† Department of Physical Sciences,
P.O. Box 3000, FIN-90014 University of Oulu, Finland
‡ Università di Firenze and INFN Sezione di Firenze,
Via G. Sansone 1, I-50019 Sesto F.no, Firenze, Italy

E-mail: † antti.keranen@oulu.fi, ‡ becattini@fi.infn.it

Abstract. Enforcing exact conservation laws instead of average ones in statistical thermal models for relativistic heavy ion reactions gives raise to so called *canonical effect*, which can be used to explain some enhancement effects when going from elementary (e.g. pp) or small (pA) systems towards large AA systems. We review the recently developed method for computation of canonical statistical thermodynamics, and give an insight when this is needed in analysis of experimental data.

Submitted to: *J. Phys. G: Nucl. Phys.*

PACS numbers: 24.10.Pa, 25.75.-q, 25.75.Dw

1. Introduction

Statistical thermal models are widely used in the analysis of relativistic heavy ion collisions (see e.g. [1, 2, 3, 4, 5], and references therein). In these models, the final stage of inelastic interactions between hadrons, the *chemical freeze-out*, is characterized by the parameters of stationary maximum entropy ensemble.

For large system, it is appropriate to choose the most straightforwardly applicable ensemble, the grand canonical (GC) one, which is parametrized by temperature T and chemical potential μ_i for each averagely conserved charge i. The GC ensemble is defined in the large volume limit, so the volume parameter V is an extensive coefficient in the expressions for such quantities as mean particle numbers. For small systems, such as pp or pA reactions, the charge conservation laws must be handled exactly, so the ensemble to be chosen is the canonical (C) one. Then, the V independent fugacities due to average conservation laws do not appear, but are replaced by *nonlinearly V dependent* canonical chemical factors, which can account for many enhancement patterns going from pp to pA to AA systems [7, 14, 11].

In this paper, we quote the newly developed computational method for relativistic B, S, Q canonical hadron ensemble, which enables us to reach previously unattainable canonical model results, up to baryon number $\mathcal{O}(100)$ whereas with old methods it was only possible to reach $B \sim 20$ [7] with very long computation times. Once this is achieved, we can give an answer to question whether the full B, S, Q canonical modelling is needed, or is the S canonical (B, Q grand canonical) or GC approximation accurate enough.

2. Methods

In order to pass from the usual grand-canonical partition function Z_{GC} to the one fulfilling exact internal symmetries, we employ a well-known group theoretical method, first introduced by Cerulus [8, 9, 10]. By denoting the set of conserved quantum numbers by $\{C_i\}$, the canonical partition function $Z_{\{C_i\}}$ can be obtained by using a projecting operator onto the conserved quantum numbers. In the case of N internal symmetries of type $U(1)$, the projection takes the form:

$$Z_{\{C_i\}}(T,V) = \left[\prod_{i=1}^{N} \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\phi_i e^{-\mathrm{i}C_i\phi_i}\right] Z_{GC}(T,V,\{\lambda_{C_i}\}). \tag{1}$$

where $\phi_i \in [0, 2\pi)$ is a $U(1)$ group parameter and a Wick-rotated fugacity factor $\lambda_{C_i} = e^{i\phi_i}$ is introduced, for every charge C_i.

In heavy ion reactions, the relevant set of conserved charges is $\{C_i\} = B, S, Q$, namely baryon number, strangeness and electric charge. Applying this set to the previous equation yields a triple integral, which is computationally extremely time consuming for $B \gtrsim 5$. In [11] we have eliminated analytically the baryon integration, and have obtained a very efficient way to calculate the full canonical (B, S, Q) hadron thermodynamics. The applied form of partition function reads:

$$\begin{aligned} Z_{B,S,Q}(T,V) = & \frac{Z_0}{(2\pi)^2} \int_0^{2\pi} \mathrm{d}\phi_S \int_0^{2\pi} \mathrm{d}\phi_Q \\ & \times \cos\left(S\phi_S + Q\phi_Q + B \arg\omega(\phi_S,\phi_Q)\right) \\ & \times I_B(2|\omega(\phi_S,\phi_Q)|) \end{aligned} \tag{2}$$

$$\times \ \exp\left[2\sum_M z_i^1 \cos(S_i\phi_S + Q_i\phi_Q)\right],$$

where Z_0 denotes the partition function for hadrons carrying no relevant charge and I is the modified Bessel function. The sum is over mesons, and z_i^1 is the one-particle partition function. The ω above is defined as $\sum_B z_i^1\, e^{\mathrm{i}(S_i\phi_S+Q_i\phi_Q)}$, where the summation runs over baryons but not antibaryons. Now, we are left with a double integration only which can be then performed numerically with no major problem.

The mean particle numbers of primary hadrons (i.e. those directly emitted from the hadronizing source) $\langle N_i\rangle$ are obtained from the equation (3) [12, 6] by taking the derivative of the canonical partition function with respect to a fictitious fugacity λ_i:

$$\begin{aligned}\langle N_i\rangle =&\ \lambda_i \frac{\partial \ln Z_{B,S,Q}(T,V)}{\partial\lambda_i}\bigg|_{\lambda_i=1}\\ =&\ \frac{Z_{B-B_i,S-S_i,Q-Q_i}(T,V)}{Z_{B,S,Q}(T,V)} z_i^1,\end{aligned} \tag{3}$$

where the quantity $Z_{B-B_i,S-S_i,Q-Q_i}/Z_{B,S,Q}$ is called *chemical factor* [6]. In the large volume (thermodynamical) limit, this expression becomes the GC one, i.e. $\langle N_i\rangle = \lambda_B^{B_i}\lambda_S^{S_i}\lambda_Q^{Q_i} z_i^1$.

3. Numerical results

In this paper, we perform no actual fitting procedure to the experimental data, but only use some suggestive thermal parameter values from our previous work [1]. This is because our main motivation is to show systematically the canonical effects in different systems in order to give the reader an insight on applicability of different thermal approaches, namely full canonical, strangeness canonical and grand canonical.

In the following, we fix the temperature and baryon density in several cases. Strangeness and charge to baryons ratio are fixed according to initial nuclear composition. These conditions lead to fixed fugacities and particle ratios in GC formalism. GC approximations are then to be compared with full canonical and S-canonical counterparts, which are nonlinearly dependent on net baryon content.

In figure 1 we show the canonical kaon enhancement in the conditions relevant to GSI energies, 1.7 GeV per nucleon in SIS AuAu collisions. The theoretical production ratio K^+ over number of participating nucleons is shown to point out the canonical strangeness effect. Note that we have been able to compute the full chemical factor even for $B = 400$. The ratio increases very slowly towards the GC limit, thus strangeness must always be handled canonically when analysing SIS results. The relative difference between full canonical and strangeness-canonical results is 13% for $B = 2$, decreasing to 4% for $B = 10$ and to 1% for $B = 40$. It must be pointed out that the above results are calculated using an isospin symmetric initial configuration whereas $Z/A \simeq 0.4$ for gold nucleus. However, this variation essentially gives no change on the relative differences above, although the ratio $\mathrm{K}^+/N_{\mathrm{part}}$ decreases naturally due to the initial neutron excess.

In Au–Au collisions at AGS energies (11.6 A GeV momentum), the chemical freeze-out temperature is found to be around $T = 120$ MeV and the baryon density close to the normal nuclear density [1]. In figure 2 we plot the calculated $\mathrm{K}^+/N_{\mathrm{part}}$ with results obtained in peripheral to central Au–Au, Si–Au and Si–Al collisions. It can be seen that all the way up from $B = 2$ to $B = 60$ the full canonical and strangeness-canonical results are essentially the same. All central reaction results lie on the region where the canonical effects are negligible and the GC formalism applies. Strangeness enhancement in Au–Au system does not definitely look like being of canonical origin, whereas Si–Au and Si–Al follow roughly the curve at the normal nuclear density (not shown in the figure). The above argument also applies for the $\mathrm{K}^-/N_{\mathrm{part}}$ enhancement pattern [11].

The best fit temperature for the multiplicities measured by NA49 experiment at SPS in central Pb–Pb reactions at 158 A GeV is found to be $T \sim 160$ MeV [1]. In figure 3 we show the strange baryon enhancement by using a baryon density 0.3 fm^{-3}. Although the baryon density in our analysis [1] was found to be $n_B \sim 0.2$ fm^{-3} this larger value is used in order to probe the largest reasonable canonical effect. All results in figure 3 are normalized to the baryon multiplicities per participant at the point $N_{\mathrm{part}} = 2$. This choice reveals the slight difference between the results obtained using the S-canonical approximation and the B, S, Q-canonical calculation. An interesting feature here is the fact, that the S-canonical approximation leads to an overestimation of the canonical effect.

Acknowledgments

We would like to acknowledge stimulating discussions with Esko Suhonen, Jean Cleymans, Krzysztof Redlich and many other colleagues participating the conference Strangeness in Quark Matter 2001, and especially the organizers of that pleasant meeting. One of us (A.K.) acknowledges the support of the Bergen Computational Physics Laboratory in the framework of the European Community - Access to Research Infrastructure action of the Improving Human Potential Programme.

References

[1] F. Becattini, J. Cleymans, A. Keränen, E. Suhonen and K. Redlich, *Phys. Rev.* C64 (2001) 024901
[2] J. Cleymans, D. Elliott, R.L. Thews and H. Satz, *Z. Phys.* C74 (1997) 319
[3] P. Braun-Munzinger, I. Heppe and J. Stachel, *Phys. Lett.* B465 (1999) 15
[4] J. Cleymans and K. Redlich, *Phys. Rev. Lett.* 81 (1998) 5284
[5] J. Cleymans, D. Elliott, A. Keränen, E. Suhonen, *Phys. Rev.* C57 (1998) 3319
[6] F. Becattini and U. Heinz, *Z. Phys.* C76 (1997) 269
[7] J. Cleymans, A. Keränen, M. Marais and E. Suhonen, *Phys. Rev.* C56 (1997) 2747
[8] F. Cerulus, *Nuovo Cimento* 19 (1961) 528
[9] K. Redlich and L. Turko, *Z. Phys.* C5 (1980) 201
[10] L. Turko, *Phys. Lett.* B104 (1981) 153
[11] A. Keränen and F. Becattini, e-Print: nucl-th 0112021
[12] R. Hagedorn and K. Redlich, *Z. Phys.* C27 (1985) 541
[13] L. Ahle *et al.* (E-802 collaboration), *Phys. Rev.* C60 (1999) 044904
[14] K. Redlich, S. Hamieh and A. Tounsi, *J. Phys. G: Nucl. Part. Phys.* 27 (2001) 413

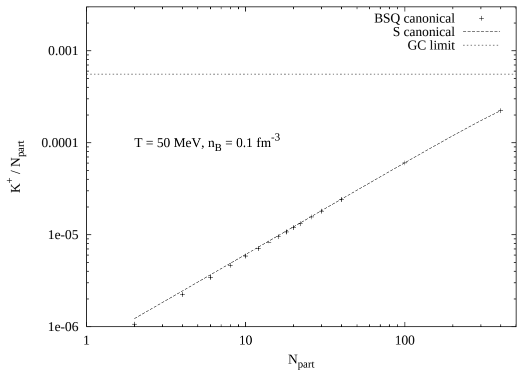


Figure 1. Canonical enhancement of kaons as a function of number of participants in the conditions relevant to GSI energies.

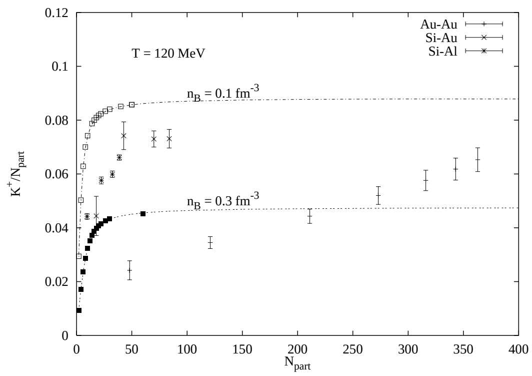


Figure 2. Theoretical $K^+/N_{\rm part}$ curves at fixed temperature for two different baryon densities shown along with AGS experimental results [13]. Curves are strangeness canonical while squares are full canonical results

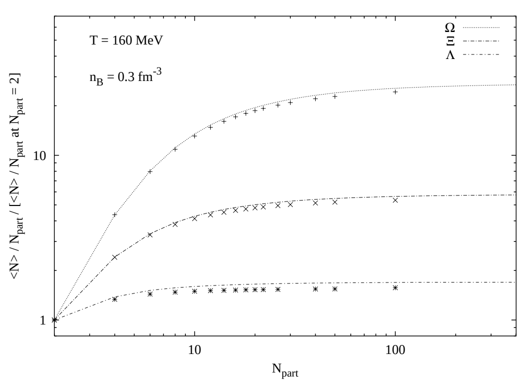
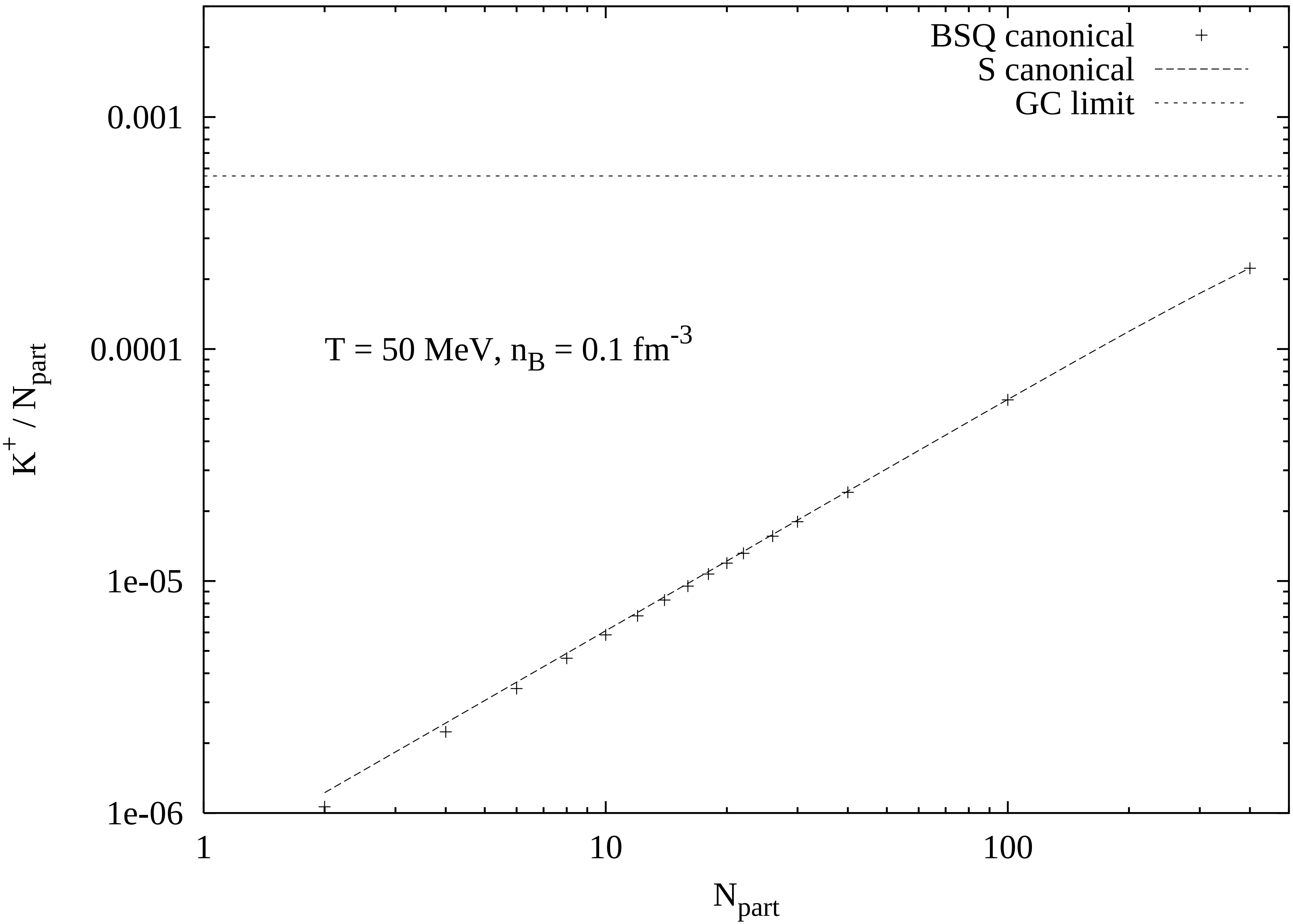
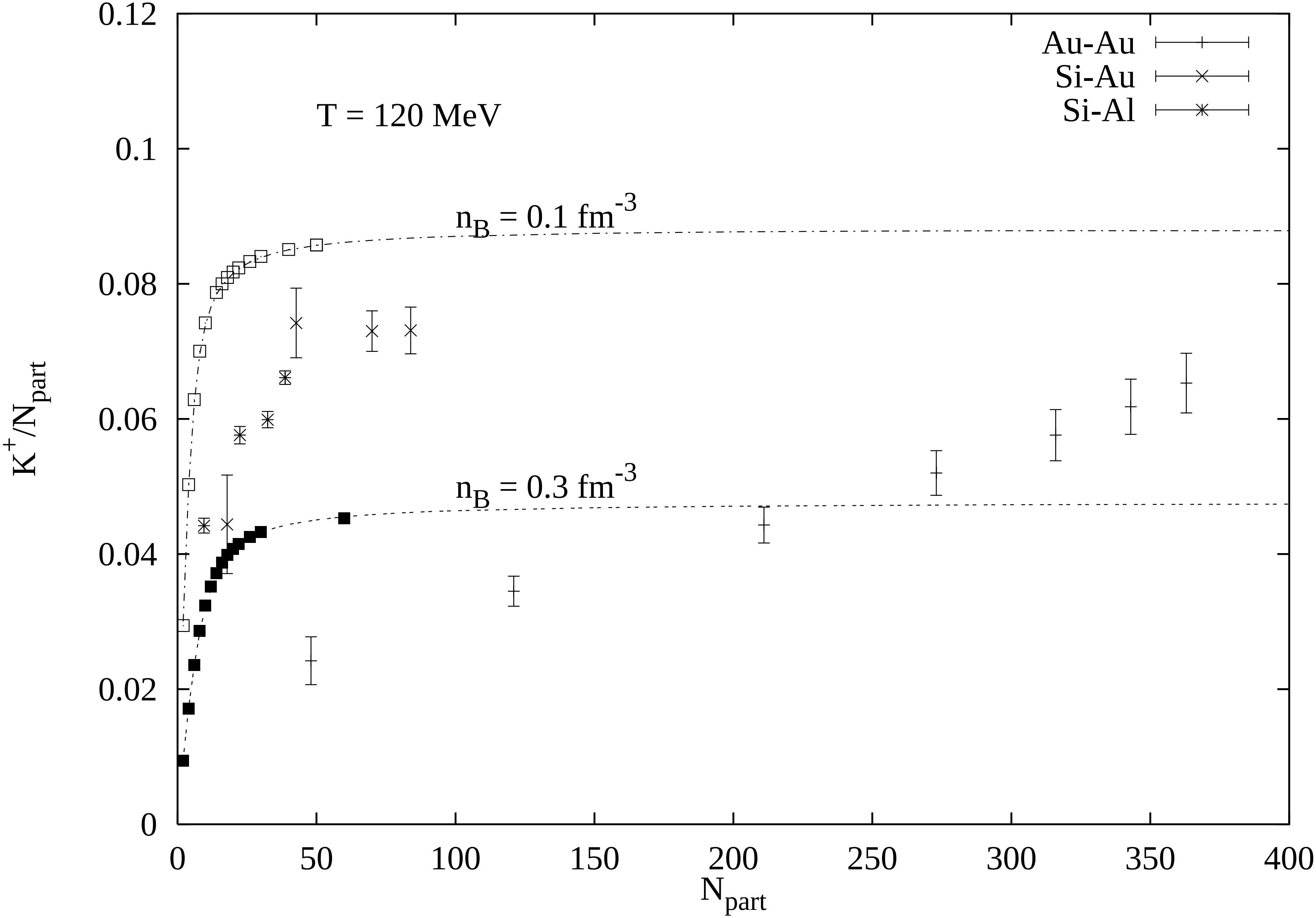


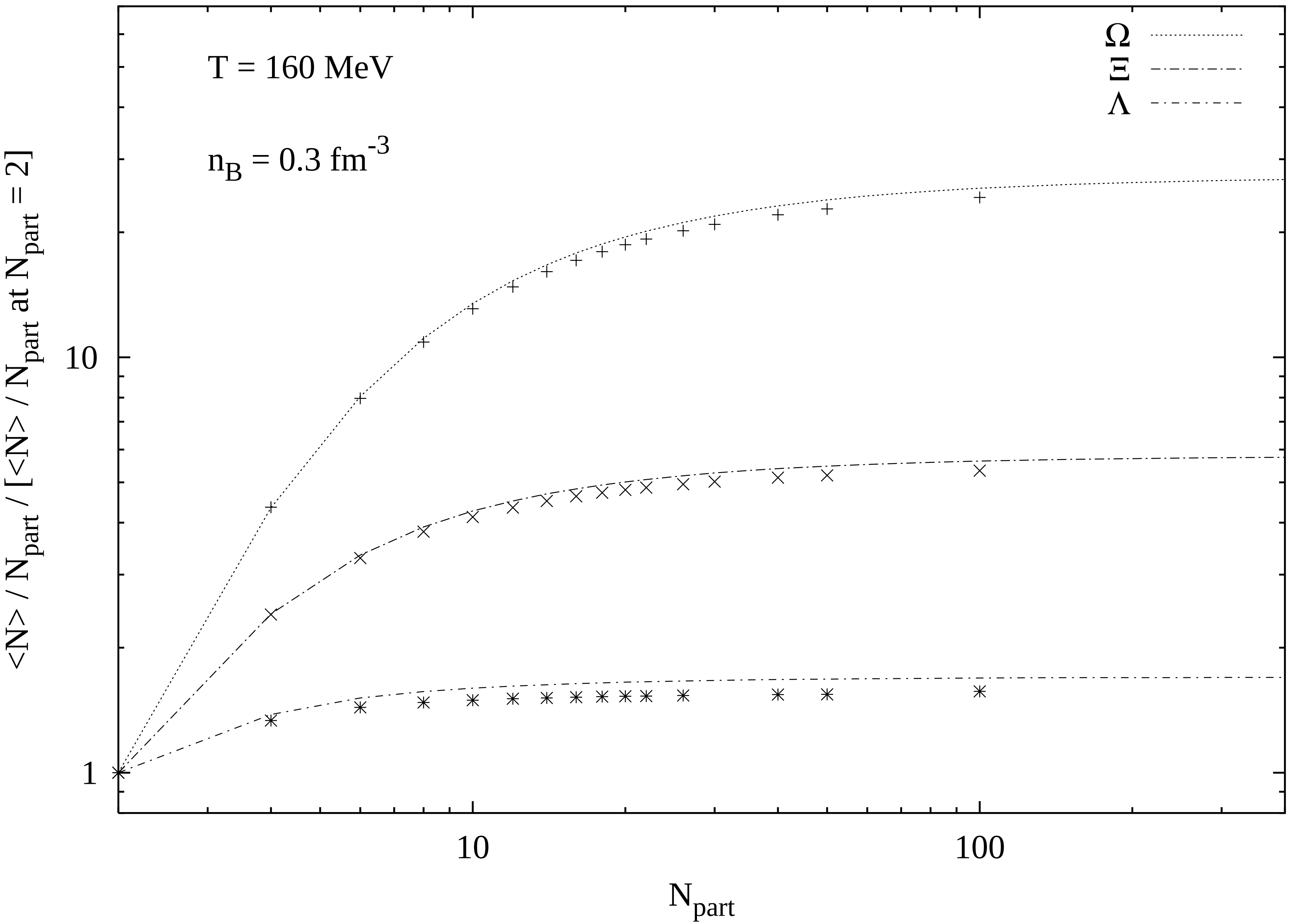
Figure 3. Strange baryon enhancement in the conditions relevant to the SPS energies. Hadron multiplicities are normalized to results at $B = 2$. Curves are strangeness canonical while crosses are full canonical results.



BSQ canonical
S canonical
GC limit
T = 50 MeV, n_B = 0.1 fm^{-3}
0.001
0.0001
1e-05
1e-06
1
10
100
K^+ / N_{part}
N_{part}



0.12
0.1
0.08
0.06
0.04
0.02
0
0
50
100
150
200
250
300
350
400
K^+/N_{part}
N_{part}
Au-Au
Si-Au
Si-Al
T = 120 MeV
$n_B = 0.1\ fm^{-3}$
$n_B = 0.3\ fm^{-3}$



T = 160 MeV
$n_B = 0.3\ fm^{-3}$
Ω
Ξ
Λ
$\langle N \rangle / N_{part}$ / [$\langle N \rangle / N_{part}$ at N_{part} = 2]
N_{part}
1
10
10
100